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According to TL/IACS CSR Part 1/Chapter 1/Section 2 /para 3.2, in case of ships having a rule length over 350m, the Society should have its own procedure for the calculation of the wave loads. The aim of the present procedure is therefore to provide methodological guidance of TL for the calculation of the wave loads for oil tankers and bulk carriers with a rule length over 350 m. As the procedure provides generic guidance on determination of wave loads, it can also be used for the calculation of the wave loads for other type and length of ships.

TÜRK LOYDU



PROCEDURE FOR DESIGN WAVE LOADS ON SHIP STRUCTURES

August 2021

This latest edition incorporates all rule changes. The latest revisions are shown with a vertical line. The section title is framed if the section is revised completely. Changes after the publication of the rule are written in red colour.

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INTRODUCTION

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A. General

The aim of the present procedure is to provide methodological guidance for the calculation of the wave loads on oceangoing vessels, including vessels with rule length over 350m.

This procedure is based on the principles of marine hydrodynamics and is based on the following assumptions which are broadly acceptable in the international literature and whose adoption is necessary for the simplification of the ship-wave interaction problem:

- The sea water is considered as inviscid and incompressible fluid.
- The flow around the ship is considered irrotational.
- The equations of ship motions are linearized.
- The ocean waves are considered as an ergodic and stationary stochastic process.

The procedure gives guidance for the calculation of the ship responses (including pressures and wave loads) in both, regular and irregular waves. For the case of irregular waves, the short-term and the long-term responses are produced, the latter being necessary for ship-design purposes.

B. Background

According to TL CSR Part 1/Chapter 1/Section 2 /para 3.2, in case of ships having a rule length over 350m, the Society should have its own procedure for the calculation of the wave loads, instead of using the relevant methodology included in the Rules and which is applicable only to ships up to this length.

The formulae currently in use by the Classification Societies for the calculation of the wave loads are the result of a variety of statistical analyses of theoretical and experimental studies and full-scale measurements, and they have been proven adequate for ships having a length less than 350m. For ships of length greater than 350 m the use of the conventional calculation of applied wave loads needs additional verification due to the lack of adequate accumulated experience for this size of vessels.

The aim of the present procedure is to provide the necessary methodological guidance to ship designers for the calculation of the wave loads on any oceangoing vessel, including vessels having a rule length over 350m.

The ship responses in a real sea state is a stochastic process and the probability theory is a power tool to assess design values of ship motions and the associated wave-induced loads. In the context of the ship design process, values of wave loads that have a specified pre-defined probability of non-exceedance during the ship's lifetime, are required. For linear wave induced responses, theoretical methods to estimate short- and long-term probability distributions are well established. For nonlinear response, it is difficult to obtain sufficiently accurate extreme value estimates. However, the extreme nonlinear response is usually related in time to the extreme linear response. In this aspect, linear approach will be also adopted in this procedure.

C. Basic Definitions

For the sake of clarity and for the better understanding of the present procedure, following definitions are given:

- 1.1 **Regular wave** is called a two-dimensional (long-crested) wave of sinusoidal shape, described by the ideal fluid potential flow.
- 1.2 **Irregular waves** are non-symmetric waves, where the phase velocity depends on wave height.
- 1.3 **Wave period T** is the time interval between successive crests of a wave sequences (s).
- 1.4 **Wave frequency f** is the inverse of the wave period (s^{-1}).
- 1.5 **Wave angular frequency ω** is the angular frequency given by the relation $\omega=2\pi/T$ (rad/s).
- 1.6 **Wave amplitude A** is called the distance of a wave crest or wave trough from the undisturbed free surface (m).
- 1.7 **Wave height H** is the distance between a successive crest and trough (m).
- 1.8 **Significant wave height $H^{1/3}$** is the average of the 1/3 highest wave heights (m).
- 1.9 **Wave length λ** is the distance between two successive crests or troughs (m).
- 1.10 **Wave number k** is defined as the ratio $2\pi/\lambda$
- 1.11 **Peak period T_p** , is the wave period or the period of another response e.g heave motion, with most energy in the wave or the response spectrum (s).
- 1.12 **Zero-up-crossing period T_z** is the wave or response period, in seconds, between two up-crossings of the undisturbed mean level (s).
- 1.13 **Phase velocity** is the propagation velocity of the wave form (m/s).
- 1.14 **Probability density function $p_X(x)$** is a function specifying that the probability that X will lie within a specific portion of x.

$$\text{Prob}[x \leq X \leq x+dx] = p_X(x)dx$$

It is evident that the area under the probability density function equals to 1.

- 1.15 The cumulative probability distribution function, $P_X(x)$, often referred to as simply the “**probability distribution**” is the below integral of the probability density function

$$P_X(x) = \int_{-\infty}^x p_X(x)dx$$

- 1.16 The **average** or the **expected** value, either of a random variable X or, more generally, of any function of X, $f(x)$, is defined as follows:

$$E[X] = \int_{-\infty}^{\infty} xp_X(x)dx \quad \text{and} \quad E[f(X)] = \int_{-\infty}^{\infty} f(x)p_X(x)dx$$

- 1.17** The mean square of a random variable X, is the expected value of X^2 , which is the second moment of $p_X(x)$, taken about $x = 0$.

$$E[X^2] = \int_{-\infty}^{\infty} x^2 p_X(x) dx$$

- 1.18** **Variance** σ^2 is defined to be the expected value of the square of the deviation from the mean:

$$\sigma^2 = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 p_X(x) dx$$

It can be proved that $\sigma^2 = E[X^2] - \mu^2$

- 1.19** The square root σ of the variance is called **standard deviation**.

- 1.20** **Gaussian distribution** is called the following probability distribution which is defined only by means of its mean and its variance:

$$p_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < +\infty$$

- 1.21** **Rayleigh distribution** is the probability distribution defined by the following relation:

$$p_X(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \quad 0 < x < +\infty$$

The surface elevation X at any point in the ocean is a random variable with a Gaussian distribution and a zero mean. The peak values of X, denoted here as \underline{X} , have a Rayleigh probability density function.

- 1.22** The **covariance** C_{XY} of the random variables X and Y is defined as:

$$C_{XY} = E[(X - \mu_X)(Y - \mu_Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \mu_X)(y - \mu_Y) p_{XY}(x, y) dx dy = E[XY] - E[X]E[Y]$$

When $X = Y$, the covariance C_{XX} reduces to the variance σ_X^2 of the random variable X.

- 1.23** A random process is called **stationary**, if its statistical characteristics do not change with time t. It has been found experimentally that, in the short term sense, the sea surface elevation is a stationary random process.

- 1.24** A random process is said to be **ergodic** if its statistical properties can be deduced from a single, sufficiently long in time, random sample of the process. Obviously, an ergodic process must be stationary, but a stationary random process is not necessarily ergodic. Ocean waves are considered as ergodic process.

- 1.25** The **auto-correlation function**, denoted by $R(\tau)$, is the average, or expected value, of the product of any two values of X: $X_1 = X(t_1)$ and $X_2 = X(t_2) = X(t_1 + \tau)$, and it is defined as follows:

$$R(\tau) = E[X(t)X(t + \tau)] = E[X_1 X_2] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 p_{XX}(x_1, x_2) dx_1 dx_2$$

The auto-correlation function is evaluated as the product of two readings taken from the same record where the reading points are separated by a shift in time t . For a stationary process, $R(\tau)$ is independent of time t .

1.26 Spectral density function $S(\omega)$ (or simply spectrum) is called the Fourier transform of the autocorrelation function $R(\tau)$.

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R(\tau) \cos(\omega\tau) d\tau \quad \text{and} \quad R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) \cos(\omega\tau) d\omega$$

Since functions $S(\omega)$ and $R(\tau)$ are both real and non-negative functions, when we are referred to physical problems (like ocean waves) where the negative time and frequency are meaningless, we can use the following relation:

$$R(\tau) = \int_0^{+\infty} 2S(\omega) \cos(\omega\tau) d\omega$$

The area $S(\omega_0)\delta\omega$ within a bandwidth $\delta\omega$ is directly proportional to the total energy of all of the components that lie within the band $(\omega_0 - 1/2 \cdot \delta\omega, \omega_0 + 1/2 \cdot \delta\omega)$. For this reason, the Spectral density function $S(\omega)$ is often called energy spectrum or wave spectrum.

One of the principal advantages of the spectral density function is that all basic characteristics of a random process can be expressed in terms of moments of this function:

$$m_n = \int_0^{+\infty} \omega^n S(\omega) d\omega$$

where n may be any integer.

1.27 Narrow-Band Process is called a process that is made up of components whose frequencies lie within a narrow band or range, whose width is small compared with the magnitude of the center frequency of the band ω_0 . Analysis of ocean wave data showed that for a fully developed, wind-generated, mid-ocean sea state the wave spectrum is relatively narrow-banded.

SECTION 2**LINEAR SHIP RESPONSES TO REGULAR WAVES**

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A. Frequency of Encounter

Consider a ship advancing at a steady mean forward speed U in a train of regular waves of small amplitude moving in six degrees of freedom. The angle β measured between the direction of U and the direction of wave propagation, defines the ship's heading ($\beta = 180^\circ$ corresponds to head seas). It is assumed that both the wave excitation forces and the resultant oscillatory motions are linear and harmonic, acting at the frequency of encounter, ω_e , expressed as follows:

$$\omega_e = \left| \omega_o - \frac{\omega_o^2}{g} U \cos \beta \right|$$

where ω_o is the circular frequency of the incident waves, and g is the acceleration of gravity.

For the computation of all responses of any vessel to regular waves, it is necessary to deal with the complete motions of a ship with six degrees of freedom, considering important couplings among them. The linear equations will be presented for a ship advancing at constant mean forward speed with arbitrary heading in a train of regular sinusoidal waves.

Detailed derivation of the equations can be found in Salvensen et al (1970), Newman (1977), Ogilvie (1964) and Wehausen (1971).

B Equations of Motion

The linearization of the equations is based on the assumption of small ship motions (which is applicable to stable ships sailing in relatively small amplitude waves).

The following right handed coordinate system (x,y,z) will be used, which is fixed to the mean floating position of the body and with the positive z -axis directed vertically upwards through the center of gravity of the ship. The origin of the system is located on the undisturbed free-surface and the xz plane coincides with the plane of symmetry of the ship. The Ox axis is directed along the vessel and the Oy is directed portside. The unit vectors along Ox , Oy and Oz axes are denoted by \mathbf{i} , \mathbf{j} and \mathbf{k} respectively.

Let η_1 , η_2 and η_3 respectively denote the surge, sway and heave motions of the ship. Furthermore let η_4 , η_5 and η_6 denote the roll, pitch and yaw angles respectively.

The motion of any point on the ship is written as:

$$\mathbf{s} = \eta_1 \mathbf{i} + \eta_2 \mathbf{j} + \eta_3 \mathbf{k} + \boldsymbol{\omega} \times \mathbf{r} \quad , \quad (2.2.1)$$

where

$$\boldsymbol{\omega} = \eta_4 \mathbf{i} + \eta_5 \mathbf{j} + \eta_6 \mathbf{k} \quad \text{and} \quad \mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \quad , \quad (2.2.2)$$

By replacing following formula is obtained:

$$\mathbf{s} = (\eta_1 + z\eta_5 - y\eta_6) \mathbf{i} + (\eta_2 - z\eta_4 + x\eta_6) \mathbf{j} + (\eta_3 + y\eta_4 - x\eta_5) \mathbf{k} \quad , \quad (2.2.3)$$

Abkowitz (1969) has shown that for a ship with a vertical longitudinal plane of symmetry the system of linearized equations is written in the form:

$$\begin{aligned} \Delta(\ddot{\eta}_1 + z_c \ddot{\eta}_5) &= F_1 \\ \Delta(\ddot{\eta}_2 - z_c \ddot{\eta}_4 + x_c \ddot{\eta}_6) &= F_2 \end{aligned}$$

$$\begin{aligned}
 \Delta(\ddot{\eta}_3 - x_c \ddot{\eta}_5) &= F_3 \\
 I_{44} \ddot{\eta}_4 - I_{46} \ddot{\eta}_6 - \Delta z_c \ddot{\eta}_2 &= F_4 \\
 I_{55} \ddot{\eta}_5 + \Delta[z_c \ddot{\eta}_1 - x_c \ddot{\eta}_3] &= F_5 \\
 I_{66} \ddot{\eta}_6 - I_{64} \ddot{\eta}_4 + \Delta x_c \ddot{\eta}_2 &= F_6
 \end{aligned} \tag{2.2.4}$$

where:

$F_j(t)$, $j=1,2,3$ are the total forces in the x,y,z directions, respectively.

$F_j(t)$, $j=4,5,6$ are the total moments acting about the x,y,z axes, respectively.

Δ , is the vessel's displacement (total mass).

I_{ij} , $j=4,5,6$ are the moments of inertia around the x,y,z axes, respectively.

$I_{46}=I_{64}$, is the roll-yaw product of inertia.

$(x_c, 0, z_c)$ are the coordinates of the center of gravity of the ship.

$\ddot{\eta}_j(t)$ is the acceleration in the j-th degree of freedom, where $j=1,2,3,4,5,6$ refers to surge, sway, heave, roll, pitch and yaw, respectively

The linearized equations of motions may be also written in the form:

$$\sum_{k=1}^6 \Delta_{jk} \ddot{\eta}_k = F_j(t) = F_{Gj} + F_{Hj}, \quad j = 1,2,3, \dots, 6 \tag{2.2.5}$$

where

F_{Gj} is the component of the gravitational force acting on the vessel in the j-th direction,

F_{Hj} is the component of the fluid force acting on the vessel in the j-th direction,

Δ_{jk} is the inertia matrix

$$\Delta_{jk} = \begin{bmatrix} \Delta & 0 & 0 & 0 & +\Delta z_c & 0 \\ 0 & \Delta & 0 & -\Delta z_c & 0 & +\Delta x_c \\ 0 & 0 & \Delta & 0 & -\Delta x_c & 0 \\ 0 & -\Delta z_c & 0 & I_{44} & 0 & -I_{46} \\ +\Delta z_c & 0 & -\Delta x_c & 0 & I_{55} & 0 \\ 0 & +\Delta x_c & 0 & -I_{46} & 0 & I_{66} \end{bmatrix} \tag{2.2.6}$$

Since only the vessel's response to sinusoidal waves is being considered in this section, the time dependent ship responses can be written in the form:

$$\eta_j(t) = \bar{\eta}_j e^{i\omega_e t}, \quad j=1,2,\dots,6 \tag{2.2.7}$$

Where ω_e is the frequency of encounter:

$$\omega_e = \omega_0 - \frac{\omega_0^2}{g} U_0 \cos\beta$$

In this relation, ω_0 is the circular frequency of the incident waves, g is the acceleration of gravity, and, $\bar{\eta}_j$ is the complex amplitude of the vessel's response in the j-th direction.

The gravitational forces are simply due to the weight of the vessel applied at the center of gravity. Since the mean gravitational forces cancel the mean buoyant forces, they are usually combined with the hydrostatic part of the fluid flow to give the net hydrostatic forces.

The fluid (hydrostatic and hydrodynamic) forces acting on the ship are obtained by integrating the fluid pressure over the wetted part of the hull. The components of the fluid forces acting in each of the six degrees of freedom are given by the formula:

$$F_{Hj} = \iint_S P \cdot n_j \cdot ds, \quad j = 1, 2 \dots 6, \quad (2.2.8)$$

where:

- n_j is the generalized normal vector to the hull surface,
- P is the fluid pressure,
- S is the wetted hull surface

The components of the generalized normal are given by the following relations:

$$\begin{aligned} \vec{n} &= (n_1, n_2, n_3) \\ \vec{r} \times \vec{n} &= (n_4, n_5, n_6) \end{aligned}$$

Where \vec{n} is the unit normal to the hull surface out of the fluid and \vec{r} the position vector to a point of the hull

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

The pressure on the body can be found by using the Bernoulli's equation for inviscid and irrotational flow:

$$P = \frac{1}{2}\rho U_0^2 - \rho \frac{\partial \Phi}{\partial t} - \frac{1}{2}\rho(\nabla\Phi \cdot \nabla\Phi) - \rho g z, \quad (2.2.9)$$

Where $\Phi(x,y,z)$ is the flow potential, a scalar function the gradient of which gives the fluid velocity.

The fluid forces can be decomposed to a hydrostatic (F_{HSj}) and hydrodynamic (F_{HDj}) part as follows:

$$F_{Hj} = F_{HSj} + F_{HDj}, \quad (2.2.10)$$

Hydrostatic component:

$$F_{HSj} = -\rho g \iint_S z \cdot n_j \cdot ds \quad (2.2.11)$$

Hydrodynamic component:

$$F_{HDj} = -\rho \iint_S \left(\frac{1}{2}U_0^2 - \frac{\partial \Phi}{\partial t} - \frac{1}{2}(\nabla\Phi \cdot \nabla\Phi) \right) n_j ds \quad (2.2.12)$$

a) Net hydrostatic forces

As previously defined the net hydrostatic forces F_{HSj}^* are the sum of the gravitational and hydrostatic forces

$$F_{HSj}^* = F_{HSj} + F_{Gj}, \quad j = 1, 2, \dots 6, \quad (2.2.13)$$

After the calculation of the integrals the net hydrostatic forces are written in the form:

$$F_{HSj}^* = -\sum_{k=1}^6 C_{jk} \bar{\eta}_k e^{i\omega_e t} \quad (2.2.14)$$

Where C_{jk} are the hydrostatic restoring force coefficients and $\bar{\eta}_k e^{i\omega_e t}$ replaces the arbitrary motions $\eta_k(t)$.

The hydrostatic restoring force coefficient C_{jk} gives the net hydrostatic force acting in the j -th direction due to a unit displacement in the k -th direction. The hydrostatic restoring force coefficients C_{jk} are obtained by the following relations:

$$\begin{aligned}
 C_{jk} &= 0, \text{ except for the following values} \\
 C_{33} &= \rho g \int B(x) dx \\
 C_{35} &= C_{53} = -\rho g \int x B(x) dx \\
 C_{44} &= \rho g \nabla GM_T \\
 C_{55} &= \rho g \int x^2 B(x) dx + \rho g \nabla (KB - KG)
 \end{aligned} \tag{2.2.15}$$

where:

- $B(x)$ is the sectional waterline beam,
- GM_T , is the transverse metacentric height of the ship
- KB , is the vertical distance of the ship's center of gravity from the base line
- KG , is the vertical distance of the ship's center of gravity from the base line.

In the relations (2.2.15) all integrals are taken over the length of the ship.

b) Hydrodynamic forces

In the context of the linearization, the following decomposition to the velocity potential is considered:

$$\begin{aligned}
 \Phi(x, y, z; t) &= [-U_0 x + \varphi_S(x, y, z)] + \varphi_T e^{i\omega_e t} = [-U_0 x + \varphi_S(x, y, z)] + \\
 &+ [\varphi_I + \varphi_D + \sum_{j=1}^6 \varphi_j \bar{\eta}_j] e^{i\omega_e t}
 \end{aligned} \tag{2.2.16}$$

where:

- φ_S is the time independent (steady) perturbation potential due to the steady translation,
- φ_T is the unsteady perturbation potential
- φ_I is the potential of the incident waves
- φ_D is the diffracted wave potential, and,
- φ_j , $j = 1, 2, \dots, 6$ are the radiation potentials due to the motion of unit amplitude in the j -th direction.

It has to be noted that in the above decomposition the time dependence of the unsteady part of the potential is included in the harmonic term $e^{i\omega_e t}$ and the potentials φ_I , φ_D and φ_j are time independent and are functions of the space variables only.

After the linearization, the hydrodynamic forces are decomposed as follows:

$$F_{HDj} = F_{EXj} + F_{RJ} = \{F_j^I + F_j^D\} e^{i\omega_e t} + \sum_{k=1}^6 T_{jk} \bar{\eta}_k e^{i\omega_e t} \tag{2.2.17}$$

where:

$$F_j^I = -\rho \iint_S n_j \left(i\omega_e - U_0 \frac{\partial}{\partial x} \right) \varphi_I ds, \quad \text{is the complex amplitude of the exciting forces due to the undisturbed wave (Froude-Krylov forces),}$$

$$F_j^D = -\rho \iint_S n_j \left(i\omega_e - U_0 \frac{\partial}{\partial x} \right) \varphi_D ds \quad \text{is the complex amplitude of the exciting forces due to the diffracted waves, and}$$

$$T_{jk} = -\rho \iint_S n_j \left(i\omega_e - U_o \frac{\partial}{\partial x} \right) \varphi_k ds$$

and the term $T_{jk}\eta_k$ is the complex amplitude of the hydrodynamic force in the j-th direction due to the forced motion in the k-th direction.

The term T_{jk} is written in the form:

$$T_{jk} = \omega_e^2 A_{jk} - i\omega_e B_{jk}, \quad (2.2.18)$$

where:

$A_{jk} = \text{Re} \left[\frac{T_{jk}}{\omega_e^2} \right]$ is the added mass in the j-th mode due to the unit motion in the k-th direction, and,

$B_{jk} = \text{Im} \left[-\frac{T_{jk}}{\omega_e} \right]$ the corresponding damping coefficient in the j-th mode due to the unit motion in the k-th direction.

Finally by introducing in the linearized equations of ship motions the expression for the several forces as expressed above, the equations of motion are written in the form:

$$\sum_{k=1}^6 [-\omega_e^2 (\Delta_{jk} + A_{jk}) + i\omega_e B_{jk} + C_{jk}] \bar{\eta}_k = F_j^I + F_j^D, \quad j = 1, 2, \dots, 6 \quad (2.2.19)$$

The above system of equations consists of six coupled linear equations for the unknown complex amplitude of the six motions $\bar{\eta}_k$.

For an arbitrarily shaped vessels the system of the six coupled equations of motion should be solved simultaneously. However, in case of a vessel which is symmetrical port and starboard, the six equations of motion may be uncoupled into two sets of three equations each. The surge, heave and pitch equations of motion are uncoupled with the sway, roll and yaw motions. This uncoupling is a consequence of the considered linearization.

For the calculation of the several terms of this system the radiation and diffraction potentials are required to be calculated by solving the corresponding hydrodynamic problems. For the solution of the hydrodynamic problem the use of the 3D Boundary Element Method (BEM) or the Finite Element Method (FEM) are recommended.

The calculated ship motions depend on the heading angle and the linear system should be solved for several heading angles. Finally, the calculated complex amplitudes are functions of the encounter frequency and the heading angle $\bar{\eta}_k(\omega_e; \beta)$ and are called Response Amplitude Operators (RAOs) or transfer functions. The concept of RAO is coming from the theory of the Linear Systems. In marine hydrodynamics, the RAO of a specific ship response to a regular (harmonic) incident wave is a complex number (which depends on the ship speed, the wave heading and the wave frequency) the modulus of which represents the amplitude of the response (which is also considered as harmonic due to the linearity) to an incident harmonic wave of unit single amplitude, while the argument of the RAO (also called "phase") represents the lag (in degrees or rads) between the incident wave and the ship response under consideration. The concept of RAO is applicable to all linear ship responses (e.g. pressure on the wetted hull, bending moments, shear forces etc) and is not restricted only to motions and accelerations. However, the RAO concept can not be used to nonlinear responses like the Von Mises stress, the expression of which involves squares of several stress components.

After the solution of the hydrodynamic problem and the determination of the several components of the potential, the pressure transfer function can be calculated from the relation (2.9) after the rejection of the non-linear term $\frac{1}{2}\rho(\nabla\phi \cdot \nabla\phi)$. Corresponding fluid forces are calculated by integrating the pressure over the wetted ship surface (see (2.2.8)).

SECTION 3**DESCRIPTION OF SEA ENVIRONMENT**

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A. The Unidirectional Wave Energy Spectrum

In an actual sea state, the wave induced response depends on the specific encountered wave system which is statistically characterized by e.g. a wave height and a wave period. Short term response is a statistical representation of the wave induced response during a specific sea state, which is defined by a wave spectrum and wave energy spreading function. Long term response is based on a statistical representation of all encountered sea states defined by a scatter diagram. These wave conditions are described in the following, and constitute necessary input to calculation of wave induced response.

A wave spectrum represents the wave energy distribution of individual wave frequencies in a stationary sea state. Wave spectra most relevant for ships are the two-parameter Pierson–Moskowitz (PM) (i.e. Bretschneider) and Jonswap wave spectra (Ochi, 1998). These are unidirectional wave spectra, referred to as single peak one-dimensional wave spectra, i.e. without wave energy spreading. In the context of the present procedure the two-parameter Pierson Moskowitz spectrum will be used for the calculation of wave loads.

The PM wave spectrum for fully developed sea is given by:

$$S(\omega) = \frac{5}{16} H_s^2 \omega_p^4 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega}{\omega_p} \right)^{-4} \right] \rightarrow S(\omega) = \frac{1}{4\pi} H_s^2 \left(\frac{2\pi}{T_z} \right)^4 \omega^{-5} \exp \left[-\frac{1}{\pi} \left(\frac{2\pi}{T_z} \right)^4 \omega^{-4} \right] \quad (3.1.1)$$

where:

H_s is the significant wave height, in m,

$\omega_p = 2\pi/T_p$, is the spectral peak frequency of the waves, in rad/sec,

T_p is the spectral peak period, in sec,

T_z is the zero cross period, in sec,

ω is the angular frequency, in rad/sec,

The zero cross period is calculated by the formula:

$$T_z = 2\pi \sqrt{\frac{m_0}{m_2}} \quad (3.1.2)$$

where m_0 and m_2 are the zeroth and second order spectrum moments respectively (see para. 1.2.26).

B. The Directional Wave Energy Spectrum

The wave energy spreading to different heading angles β describes how the wave energy is distributed over different headings relative to the main heading β_0 . A large spreading implies that the waves are short crested, while a narrow spreading implies that the waves are long crested or unidirectional. Directional short-crested wave spectra $S(\omega_e, \vartheta)$ may be expressed in terms of the unidirectional wave spectra as:

$$S(\omega_e, \vartheta) = S(\omega_e) \cdot f(\vartheta), \tag{3.2.1}$$

The angular spreading function $f(\vartheta)$, unless otherwise specified, to be taken as ([Ochi ??]):

$$f(\vartheta) = \frac{2}{\pi} \cos^2(\vartheta_0 - \vartheta), \quad \vartheta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$f(\vartheta) = 0, \quad \vartheta \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$
(3.2.2)

C. Selection of Design Sea Environment

According to TL-G 34, but TL-CSR Rules and GBS as well, the standard wave data used for the prediction of the wave loads are those related to the sea area of the North Atlantic (sea areas 8, 9, 15 and 16 in the map shown in Figure 3.3.1):

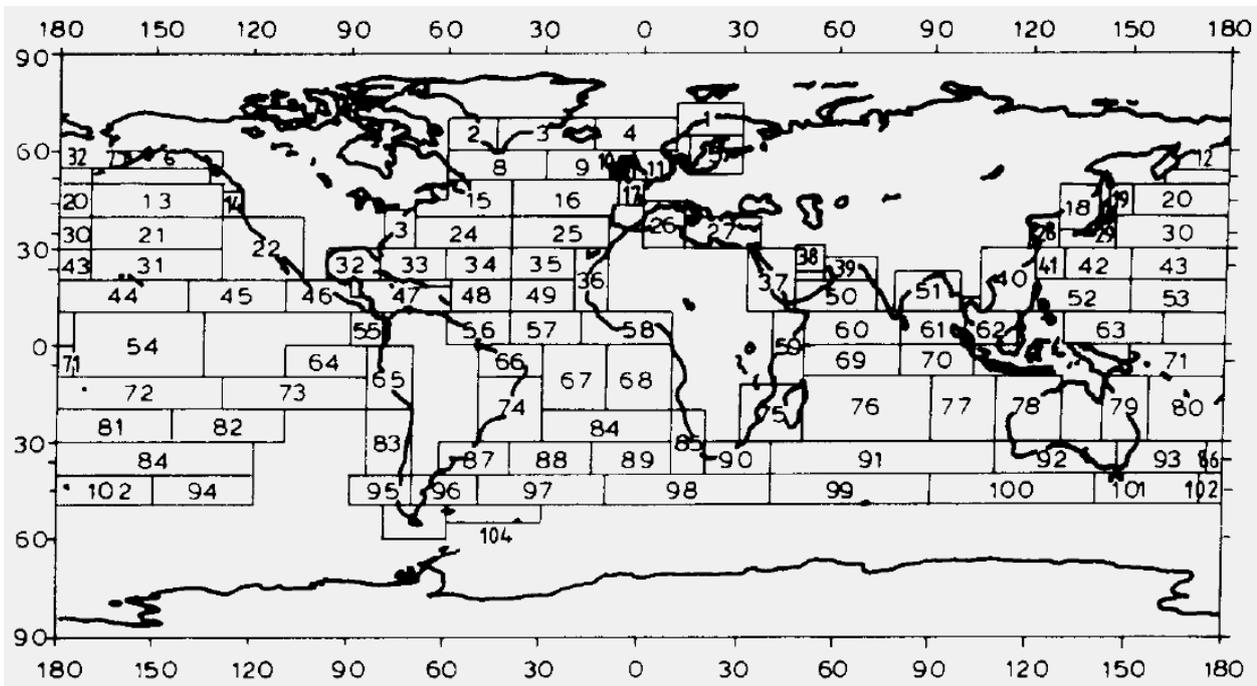


Figure 3.3.1: Definition of the extent of the North Atlantic sea area.

The scatter diagram for the North Atlantic, as per TL-G 34, is given by the following Table:

Table 1: Probability of sea-states in the North Atlantic described as occurrence per 100000 observations.
(Derived from BMT's Global Wave Statistics)

Hs/Tz	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5	17.5	18.5	SUM
0.5	0.0	0.0	1.3	133.7	865.6	1186.0	634.2	186.3	36.9	5.6	0.7	0.1	0.0	0.0	0.0	0.0	0.0	0.0	3050
1.5	0.0	0.0	0.0	29.3	986.0	4976.0	7738.0	5697.7	2375.7	703.5	160.7	30.5	5.1	0.8	0.1	0.0	0.0	0.0	22575
2.5	0.0	0.0	0.0	2.2	197.5	2158.8	6230.0	7449.5	4860.4	2066.0	644.5	160.2	33.7	6.3	1.1	0.2	0.0	0.0	23810
3.5	0.0	0.0	0.0	0.2	34.9	695.5	3226.5	5675.0	5099.1	2838.0	1114.1	337.7	84.3	18.2	3.5	0.6	0.1	0.0	19128
4.5	0.0	0.0	0.0	0.0	6.0	196.1	1364.3	3288.5	3857.5	2685.5	1275.2	455.1	130.9	31.9	6.9	1.3	0.2	0.0	13289
5.5	0.0	0.0	0.0	0.0	1.0	51.0	498.4	1602.9	2372.7	2008.3	1126.0	463.6	150.9	41.0	9.7	2.1	0.4	0.1	8328
6.5	0.0	0.0	0.0	0.0	0.2	12.6	167.0	690.3	1257.9	1268.6	825.9	386.8	140.8	42.2	10.9	2.5	0.5	0.1	4806
7.5	0.0	0.0	0.0	0.0	0.0	3.0	52.1	270.1	594.4	703.2	524.9	276.7	111.7	36.7	10.2	2.5	0.6	0.1	2596
8.5	0.0	0.0	0.0	0.0	0.0	0.7	15.4	97.9	255.9	350.6	296.9	174.6	77.6	27.7	8.4	2.2	0.5	0.1	1309
9.5	0.0	0.0	0.0	0.0	0.0	0.2	4.3	33.2	101.9	159.9	152.2	99.2	48.3	18.7	6.1	1.7	0.4	0.1	626
10.5	0.0	0.0	0.0	0.0	0.0	0.0	1.2	10.7	37.9	67.5	71.7	51.5	27.3	11.4	4.0	1.2	0.3	0.1	285
11.5	0.0	0.0	0.0	0.0	0.0	0.0	0.3	3.3	13.3	26.6	31.4	24.7	14.2	6.4	2.4	0.7	0.2	0.1	124
12.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1.0	4.4	9.9	12.8	11.0	6.8	3.3	1.3	0.4	0.1	0.0	51
13.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	1.4	3.5	5.0	4.6	3.1	1.6	0.7	0.2	0.1	0.0	21
14.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	1.2	1.8	1.8	1.3	0.7	0.3	0.1	0.0	0.0	8
15.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.6	0.7	0.5	0.3	0.1	0.1	0.0	0.0	3
16.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.2	0.2	0.1	0.1	0.0	0.0	0.0	1
SUM:	0	0	1	165	2091	9280	19922	24879	20870	12698	6245	2479	837	247	66	16	3	1	100000

According to IMO GBS FR II.1, the specified ship design life is equal to 25 years. This life span determines the number of cycles of the ship response to waves which in turn determines the probability of response exceedance per cycle. It is assumed in the numerical model determining the long-term ship responses to waves that the probability of exceeding the design value of ship response to waves is 10^{-8} , which approximately corresponds to 25 years of the specified ship design life.

SECTION 4**SHORT TERM SHIP RESPONSES**

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B. SHORT TERM STATISTICS	4- 2

A. Response Spectrum

Consistent with linear theory short term extreme values can be derived from the standard deviation σ of the response. The response spectrum can be interpreted as the energy distribution of the response with respect to wave (encounter) frequency ω_e and heading angle β . Given the RAO $\overline{\eta_k(\omega_e, \beta)}$ for a given response k (from the solution of the system 2.2.19), the response spectrum $R(\omega_e, \beta)$ is defined as:

$$R(\omega_e, \beta) = \left| \overline{\eta_k(\omega_e, \beta)} \right|^2 \cdot S(\omega_e, \beta) \quad (4.1.1)$$

The moments of the response spectrum are defined as:

$$m_{\eta, n} = \int_0^{2\pi} \int_0^{\infty} \omega^n R(\omega_e, \beta) d\omega d\beta, \quad n = 0, 1, 2, 3, \dots \quad (4.1.2)$$

where $n = 0$ provides the variance (standard deviation squared), $n = 1$ the first moment and $n = 2$ is the moment of inertia of the spectra.

B. Short Term Statistics

Short term statistics can be derived from the standard deviation σ of the response corresponding to the zero moment:

$$\sigma^2 = m_{\eta, 0} \quad (4.2.1)$$

Assuming that the response amplitudes follow the Rayleigh distribution:

$$p_X(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \quad 0 < x < +\infty, \quad (4.2.2)$$

following predictions of the statistical characteristics of the response can be made.

The probability the value of the response R to exceeds a given threshold value r_L is given by the following relation:

$$P(R > r_L) = \int_{r_L}^{\infty} \frac{r}{\sigma} \cdot e^{-\frac{r^2}{2\sigma^2}} \cdot dr = e^{-\frac{r_L^2}{2\sigma^2}} \quad (4.2.3)$$

The significant response amplitude, $\overline{R}_{1/3}$, is defined as the mean value of the highest one-third part of the amplitudes:

$$\overline{R}_{1/3} = 2\sqrt{m_{\eta, 0}} \quad (4.2.4)$$

The mean period T_1 and the average zero-crossing period T_z , in s, of the response are defined as:

$$T_1 = 2\pi \sqrt{\frac{m_{\eta, 0}}{m_{\eta, 1}}} \quad (4.2.5)$$

$$T_z = 2\pi \sqrt{\frac{m_{\eta,0}}{m_{\eta,2}}}, \quad (4.2.6)$$

The expected extreme value $E[R_{max}]$ depends on the exposure time T_d in the actual sea state. For narrow banded linear processes a good approximation is given by:

$$E[R_{max}] = \sigma \cdot \sqrt{2 \ln \left(\frac{T_d}{T_z} \right)}, \quad (4.2.5)$$

where T_z is the zero-crossing encounter period of the response. It should be noted that the probability of exceeding this value is 63%. However, the extreme value distribution is narrow for large durations and the response will usually not exceed this estimate significantly.

Another often applied metric is the extreme value, $R_{max,\alpha}$, associated with a small prescribed exceedance, probability, α . An approximate formula valid for narrow banded linear processes is:

$$R_{max,\alpha} = \sigma \cdot \sqrt{2 \ln \left(\frac{T_d}{\alpha T_z} \right)}, \quad (4.2.6)$$

SECTION 5

LONG TERM SHIP RESPONSES

Page

A. LONG TERM STATISTICS 5- 2

A. Long Term Statistics

The long term statistics of ship responses take into account the various sea conditions that the ship is expected to encounter during its life time. As a consequence, the long term statistics combine results from various short term assessments as described in the previous section.

The most common method for assessing long term statistics is to apply simple weighting of short term responses based on the probability of encountering various sea conditions. The weighting factor is defined by the probability of occurrence of each sea state as defined by the actual wave scatter diagram.

A mathematical model of the probability of exceeding a given value r_L of the ship response to waves is determined by the following expression:

$$P(R \geq r_L) = \sum_i \sum_j \sum_k p_k \cdot p_j \cdot p_i \int_{r_L}^{\infty} \int_0^{\infty} \int_0^{\infty} p_{klm}(r/(H_s, T_z)) \cdot p_{HT}(H_s, T_z) \cdot dH_s \cdot dT_z \cdot dr \quad (5.2.1)$$

where:

$p_{klm}(r/(H_s, T_z))$ is the probability density function of the random variable R, which represents any ship response to waves, in the sea state condition (HS; Tz) (see relation 4.2.2),

H_s is the significant wave height, and

T_z is the average zero up crossing wave period, at a certain angle β of ship course in relation to the wave propagation, in a given sea environmental condition, and ship loading condition;

$p_{HT}(H_s, T_z)$ is the probability density function of the occurrence of the specific sea state;

p_k is the probability of ship's heading β_k , $k=1,2, \dots, n_h$, in relation to wave direction;

p_j is the probability of ship's presence in the specific sea state $(H_s, T_z)_j$, $j=1,2, \dots, n_{ss}$, in relation to the wave direction;

p_i is the probability of ship's loading condition LC_i , $i=1,2, \dots, n_{LC}$;

r_L is a predefined threshold number.

The probability distribution function $p_{klm}(r/(H_s, T_z))$ for a specific response R follows the Rayleigh distribution and is calculated by (5.2.2), where the variance σ^2 is calculated by the corresponding response spectrum:

$$p_{klm}(r/(H_s, T_z)) = \frac{r}{\sigma^2} e^{-x^2/2\sigma^2}, \quad 0 < x < +\infty, \quad (5.2.2)$$

The probability $p_{HT}(H_s, T_z)$ of occurrence of a specific sea state with given significant wave height H_s and zero-crossing period T_z is calculated from the scatter diagram for the North Atlantic sea area given in Table 1.

According to TL-G 34, in long term calculations, all wave headings (0-360°) can be assumed to have an equal probability of occurrence. However, as per IACS document "CSR URCN1 TB Report(1) for NC01", a more realistic representation of the probability of ship's heading β_k , $k=1,2, \dots, n_h$, in relation to wave direction is proposed. The latter is adopted in this procedure for the sake of uniformity with the IACS Members. Therefore probability p_k can be calculated by the following formula:

$$\begin{aligned} p_k &= p(\beta_k; L) = a(\beta_k) \cdot (L - 90) + P(90, \beta_k), \quad 90m \leq L \leq 350m \\ p_k &= p(\beta_k; L) = 0.083333, \quad L > 350m \end{aligned} \quad (5.2.3)$$

where:

L is the ship's length in m,

$$a(\beta_k) = \frac{0.083333 \cdot P(90, \beta_k)}{260}, \text{ and}$$

$P(90, \beta_k)$ takes the values indicated in the following Table:

β_k	$P(90, \beta_k)$
0	0.271838
30	0.040681
60	0.031733
90	0.02541
120	0.051746
150	0.047857
180	0.333307
210	0.047857
240	0.051746
270	0.02541
300	0.031733
330	0.040681

For the long-term calculations a maximum spacing of 30 degrees between headings should be applied.

The probability p_j of ship's presence in the specific sea state $(H_s, T_z)_j$, $j=1,2, \dots, n_{ss}$, is normally assessed by means of the vessel's operational profile taking into account the associated wave data of the areas of operation. However, for design purposes and according to the Functional Requirement II.2 of IMO GBS, the considered sea areas are to be the North Atlantic, covering the zones 8, 9, 15 and 16 (see Figure 3.3.1) and therefore corresponding wave statistics included in TL-G 34, can be used. If we consider that the vessel sails all her life (i.e. 25 years) in the North Atlantic, the probability to sail in a specific sea state $((H_s, T_z)_j$ can be considered equal with the probability of occurrence of this stated, as can be calculated from the scatter diagram of North Atlantic (TL-G 34).

For the estimation of the probability p_i of ship's loading condition LC_i , $i=1,2, \dots, n_{LC}$ the most unfavorable Loading Conditions included in the vessel's Loading Manual shall be considered.

Taking into account equation (5.1.2) and the scatter diagram for the North Atlantic, the relation (5.1.1) for the calculation of the long-term probability that the amplitude of the ship response R will exceed the threshold value r_L , can be alternatively expressed in the following way:

$$P(R \geq r_L) = \sum_i \sum_j \sum_k \sum_h \sum_t \frac{r}{\sigma^2} e^{-x^2/2\sigma^2} \cdot p_k \cdot p_j \cdot p_i \cdot p_{ht}$$

where the probability p_{ht} , $h=1,2,\dots,17$, $t=1,2,\dots,18$ is the probability of occurrence of a specific sea state (H_h, T_t) as calculated from the scatter diagram of TL-G 34.

SECTION 6

EQUIVALENT DESIGN WAVE APPROACH

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B. DETERMINATION OF EDW CHARACTERISTICS	6-2

A. The Equivalent Design Wave concept

The Equivalent Design Wave (EDW) approach represents a consistent rational procedure that employs a direct analysis for the particular hull structure. The Equivalent Design Wave approach is widely used in the industry as an efficient engineering procedure. It is the basis of the hydrodynamic loads formulas in the different classification rules, such as the IACS Common Structural Rules.

An Equivalent Design Wave is a sinusoidal wave, or a group of waves, which is defined in order to maximize a given target (dominant) load. This target load is often a load parameter (such as vertical bending moment, acceleration, etc...), but it can be any type of ship response (local pressure, stress...).

The underlying assumption is that if the ship is investigated to resist loadings caused by selected (equivalent) regular waves, it will resist all loads expected during its lifetime.

General practice is to choose a regular sinusoidal wave for the EDW, the main advantage being its simplicity. The EDW is defined by the following parameters: frequency, heading angle, amplitude and phase angle (crest position wrt to LCG).

For each Load Case, an equivalent design wave is determined which simulates the extreme response value of the Dominant Load Response of the Load Case. The effect of the EDW for the dominant load response on other load responses is implemented through the application of appropriate Load Combination Factors.

B. Determination of EDW characteristics

The basic steps of the proposed procedure are the following:

1. Linear Seakeeping Analysis

The responses of the ship to regular waves are calculated by solving the equations of motion and the transfer functions (RAOs) are calculated, as described in Section 2, for the several target loads.

2. Short term responses

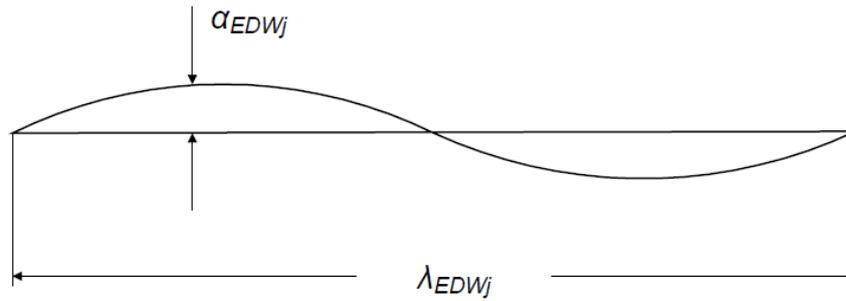
By using the calculated RAOs and the several North Atlantic input wave spectra, the corresponding response spectra and the associated response spectral characteristics (e.g. spectral moments) are calculated, as described in Section 3.

3. Long term responses

On the basis of the short term responses, a long term analysis is performed and cumulative distributions of long-term wave-induced extreme loads are calculated, as described in Section 5. In this way, by using relation (5.2.4) for each probability level the target extreme load is calculated. In the context of this procedure, and as per GBS requirements the probability level of 10^{-8} is used for strength assessment and the probability level 10^{-2} is used for fatigue assessment.

4. Equivalent design wave

These waves are characterized by wave amplitude, wave length, wave heading, and wave phase (i.e. wave crest position referenced to LCG). Where appropriate, still water loads are first added to long-term wave-induced loads.



The wave amplitude of such an equivalent extreme wave is determined by dividing the extreme long-term value of the dominant load response under consideration by the maximum value of its transfer function (RAO), occurring at the wave frequency and wave heading corresponding to the maximum amplitude of the transfer function. The wave length is obtained from the functional relationship to the wave frequency; the wave crest position, from the wave phase, as described below.

The amplitude of the equivalent regular wave can be calculated from the following formula:

$$\alpha_{EDWj} = \frac{LTEV_j}{maxRAO_j}$$

where:

α_{EDWj} , is the amplitude of the equivalent regular wave,

$LTEV_j$ is the most probable extreme value of the j-th load response corresponding to a probability of occurrence of 10^{-8} (for strength assessment) and 10^{-2} for fatigue assessment, and,

$maxRAO_j$, is the maximum value of the RAO of the j-th load response.

The equivalent wave frequency and length for each dominant load response are determined from the lifetime maximum value of the RAO of the dominant load response for each considered heading angle. When the RAO reaches maximum, the corresponding wave frequency and wave direction are denoted as equivalent wave frequency ω_{EDWj} and equivalent wave direction β . The wavelength of the equivalent regular wave can be determined by the following deepwater dispersion relation:

$$\lambda_{EDWj} = \frac{2\pi g}{\omega_{EDWj}^2}$$

where :

λ_{EDWj} , is the wavelength of the EDW for the j-th dominant load response

g , is the gravitational acceleration and

ω_{EDWj} is the frequency of the equivalent design wave

The wave crest position (phase) is calculated with respect to the LCG of the hull by the following formula:

$$x_c = -\frac{\lambda_{EDWj} \cdot \theta}{360 \cdot \cos\beta}$$

where:

x_c , is the wave crest position with respect to the LCG

θ , is the phase angle in degrees of the dominant load response, and

β , is the heading angle

For beam seas ($\beta=90^\circ$) the distance x_c is meaningless and it is not defined from the above relation. In this case the distance of the wave crest line from the vessel's center line is given by the following formula:

$$y_c = -\frac{\lambda_{EDWj} \cdot \theta}{360 \cdot \sin\beta}$$

5. Load Combination Factors (LCFs)

The effect of the EDW for a specific dominant Load response on other Load Responses is assessed through the adoption of appropriate weight coefficients called Load Combination Factors (LCFs). The LCF for any load response other than the dominant represent the relative value of the response in comparison with its long term value. The LCF is 1 or -1 for the dominant load response. When global FE analysis is performed, the load combination factors are not necessary to derive as pressures and accelerations (inertia) are directly transferred.

6. Transformation of extreme loads into nodal forces

The external pressures caused by the extreme wave-induced loads are added to inertial loads and transformed to the FE model as nodal forces.

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